Algebra II Trig Review 14.3 – 14. 6 Name Key

I. Write each expression in terms of a **single trigonometric function**. Show all **work!!**

1. $\sin\theta(\csc\theta - \sin\theta)$

$\sin\theta\left(\frac{1}{\sin\theta} - \sin\theta\right)$

$\frac{\sin\theta}{\sin\theta} - \sin^2\theta$

$1 - \sin^2\theta = \cos^2\theta$

2. $\cot\theta\sec\theta$

$\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\cos\theta}$

$\frac{1}{\sin\theta} = \csc\theta$

3. $\sin^2\theta\csc^2\theta - \tan^2\theta\cos^2\theta$

$\frac{\sin^2\theta}{\sin^2\theta}\cdot\frac{1}{\sin^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta$

$1 - \sin^2\theta$

$\cos^2\theta$

4. $\frac{\sec^2\theta - 1}{\tan\theta}$

$\frac{\tan^2\theta}{\tan\theta}$

$\tan\theta$

II. Use identities to **verify** that each statement is true. Show all work!!

5. $\sin\theta(1 + \cot^2\theta) = \csc\theta$

$\sin\theta(\csc^2\theta)$

$\frac{\sin\theta}{1}\cdot\frac{1}{\sin^2\theta}$

$\frac{1}{\sin\theta}$

$\csc\theta = \csc\theta$

6. $\frac{1}{\cos^2\theta} - 1 = \tan^2\theta$

$\frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta}$

$\frac{1-\cos^2\theta}{\cos^2\theta}$

$\frac{\sin^2\theta}{\cos^2\theta}$

$\tan^2\theta = \tan^2\theta$

III. Find the exact values using the **sum and difference formulas**.

7. $\cos 75° = \cos(30+45)$

$\cos(30)\cos(45) - \sin(30)\sin(45)$

$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \left(\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

8. $\sin 345° = \sin(300+45)$

$\sin(300)\cos(45) + \cos(300)\sin(45)$

$\left(-\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\right)$

$-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{-\sqrt{6}+\sqrt{2}}{4}$

9. $\sin 15° = \sin(45-30)$

$\sin(45)\cos(30) - \cos(45)\sin($

$\left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{}}{4}$

III. Find the exact value, use the **half-angle formulas**.

10. $\cos 165°$ (Q2) $= \cos\left(\frac{330}{2}\right)$

$\cos\left(\frac{330}{2}\right) = \pm\sqrt{\frac{1+\cos(330)}{2}}$ (Q2)

$-\sqrt{\frac{1+\sqrt{3}/2}{2}}$

11. $\sin 67.5°$ (Q1) $= \sin\left(\frac{135}{2}\right)$

$\sin\left(\frac{135}{2}\right) = \pm\sqrt{\frac{1-\cos(135)}{2}}$ (Q1)

$= \sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{2}}$

12. $\sin 105°$ (Q2) $= \sin\left(\frac{210}{2}\right)$

$\sin\left(\frac{210}{2}\right) = \pm\sqrt{\frac{1-\cos(210)}{2}}$ (Q2)

$= \sqrt{\frac{1-\left(-\sqrt{3}/2\right)}{2}}$

IV. Use the information give to find the exact value of sin 2θ and cos 2θ.

13. $90° \le \theta \le 180°$; $\sin\theta = \frac{15}{17}$ (II)

$\cos\theta = \frac{-8}{17}$

$15^2 + x^2 = 17^2$

$x^2 = 64$

$x = 8$

$\sin 2\theta = 2\sin\theta\cos\theta$

$2 \cdot \left(\frac{15}{17}\right)\left(\frac{-8}{17}\right) = \frac{-240}{289}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\left(\frac{-8}{17}\right)^2 - \left(\frac{15}{17}\right)^2 = \frac{64}{289} - \frac{225}{289} = \frac{-161}{289}$

14. $0° \le \theta \le 90°$; $\cos\theta = \frac{3}{4}$ (I)

$\sin\theta = \frac{\sqrt{7}}{4}$

$4^2 = 3^2 + x^2$

$16 = 9 + x^2$

$7 = x^2$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$2 \cdot \frac{\sqrt{7}}{4} \cdot \frac{3}{4} = \frac{6\sqrt{7}}{16} = \frac{3\sqrt{7}}{8}$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\left(\frac{3}{4}\right)^2 - \left(\frac{\sqrt{7}}{4}\right)^2$

$\frac{9}{16} - \frac{7}{16} = \frac{2}{16} = \frac{1}{8}$

V. Use the information given to find the exact value of $\sin\frac{\theta}{2}$ and $\cos\frac{\theta}{2}$.

15. $0° \le \theta \le 90°$; $\cos\theta = \frac{1}{4}$ (Q1)

$\sin\frac{\theta}{2} = \oplus\sqrt{\frac{1-\cos\theta}{2}}$

$\sqrt{\frac{1-1/4}{2}} = \sqrt{\frac{3/4}{2}}$ good

$\sqrt{3/8}$

$\cos\frac{\theta}{2} = \oplus\sqrt{\frac{1+\cos\theta}{2}}$ (Q1)

$\sqrt{\frac{1+1/4}{2}} = \sqrt{\frac{5/4}{2}}$ ok

$= \sqrt{\frac{5}{8}}$

16. $\frac{180°}{2} \le \frac{\theta}{2} \le \frac{270°}{2}$; $\sin\theta = \frac{-3}{5}$ (III)

$90° \le \frac{\theta}{2} \le 135°$ II

$\sin\frac{\theta}{2} = \oplus\sqrt{\frac{1-\cos\theta}{2}}$ (Q2)

$= \sqrt{\frac{1-(-4/5)}{2}}$ good

$= \sqrt{\frac{9/5}{2}} = \sqrt{\frac{9}{10}}$

-4, -3, 5

$\cos\theta = \frac{-4}{5}$

$5^2 = (-3)^2 + x^2$

$25 = 9 + x^2$

$16 = x^2$

$x = 4$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$= -\sqrt{\frac{1-4/5}{2}} = -\sqrt{\frac{1}{10}}$

VI. **Solve** the equation for $0 \le \theta < 360$.

17. $\sin\theta = \pm\frac{\sqrt{3}}{2}$

UNIT CIRCLE

$\theta = 60°, 120°, 240°$ and $300°$

18. $\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = 45°$ and $315°$

$360 - 45 = 315$

19. $\tan\theta = -1$

$\theta = 315°$ and $135°$

$\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$ or $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$-45°$

VII. **Solve** the equation for $0 \le \theta < 360$.

20. $2\sin^2\theta - \sin\theta - 1 = 0$

21. $\tan^2\theta = 8$

22. $2\sin\theta = \sqrt{3}$

23. $2\cos\theta\sin\theta - \cos\theta = 0$